Gauge and moduli hierarchy in a multiply warped braneworld scenario

Ashmita Das* and Soumitra SenGupta†
Department of Theoretical Physics,
Indian Association for the Cultivation of Science,
2A & 2B Raja S.C. Mullick Road,
Kolkata - 700 032, India.

Abstract

A generalized Randall sundrum model in six dimensional bulk is studied in presence of non-flat 3-branes at the orbifold fixed points. The warp factors for this model is determined in terms of multiple moduli and brane cosmological constant. We show that the requirements of a vanishingly small cosmological constant on the visible brane along with non-hierarchical moduli, each with scale close to Planck length, lead to a scenario where the 3-branes can not have any intermediate scale and have energy scales either close to Tev or close to Planck scale. Such a scenario can address both the gauge hierarchy as well as fermion mass hierarchy problem in standard model. Thus simultaneous resolutions to these problems are closely linked with the near flatness condition of our universe without any intermediate hierarchical scale for the moduli.

Introduction

Large hierarchy of mass scales between the Planck and the electroweak scales results into well-known fine tuning problem in connection with the mass of the Higgs, the only scalar particle in the standard model. It has been shown that due to large radiative corrections the Higgs mass diverges quadratically and can not be confined within Tev scale unless some unnatural tuning is done order by order in the perturbation theory. This problem has been addressed in different variants of extra dimensional models. Among all extra dimensional models the scenario proposed by Randall and Sundrum has drawn lot of attention. It assumes a warp geometry of the space-time in 5 dimensions [1]. The fifth

*E-mail address: tpad@iacs.res.in
†E-mail address: tpssg@iacs.res.in

dimension is compactified on a space S^1/Z_2 and the length scale of this extra dimension is of the order of Planck length r_c. Two 3-branes are located at the two orbifold fixed points. The exponential warping of the length scale along the fifth dimension naturally suppresses Planck scale quantities of one 3-brane, which we call hidden brane/Planck brane, into electroweak scale on the second 3-brane, which is identified as TeV-brane/visible brane and can be interpreted as our universe without introducing any new hierarchical scale into the theory. A generalization of RS model have been considered previously by introducing more than one warped extra dimension in the theory [2]. Such models have interesting implications in particle phenomenology in higher dimensional models which can be summarized as :

- Resolution of the well known fermion mass hierarchy problem among standard model fermions [3].
- The localization of massless fermions with a definite chirality on the visible 3-brane [4].
- A consistent description of a bulk Higgs and gauge fields with spontaneous symmetry breaking in the bulk, along with proper W and Z boson masses on the visible brane [5].
- Provides a stack of branes picture similar to string inspired models [2].

Motivated by the proposal of a non-vanishing cosmological constant as dark energy model for our present universe, in this work we generalize the six dimensional multiple warped model to include non-flat 3-branes. Such generalization was done earlier for five dimensional scenario in [6]. We find the warp factors solutions for both de-Sitter and anti de-Sitter 3-branes with appropriate brane tensions.

We organize our paper as follows. In the following section we explain some features of the six dimensional doubly warped model with flat 3-branes. In sec.II we describe the six dimensional doubly warped model with non vanishing cosmological constant on the 3-branes (i.e with non-flat branes). In sec.III we explain the correlation between the brane induced cosmological constant, and the ratio of the two extra dimensional moduli in the de-sitter brane. In sec.IV we present our result and show that the four important aspects, namely 1) the value of cosmological constant in the present universe 2) hierarchical warping along the two extra dimensions, 3) hierarchy between the two extra dimensional moduli and 4) the gauge hierarchy problem, have interesting correlations among themselves.

I. 6-dimensional generalization of warped geometry model

Here we briefly outline the mechanism for generalizing the 5-dimensional RS model to 6-dimensional doubly warped geometry model with flat 3-branes [2]. The 6-dimensional doubly warped model has six space-time dimensions. The

extra two spatial dimensions are orbifolded successively by Z_2 symmetry . The manifold for such geometry is $[M^{(1,3)} \times S^1/Z_2] \times S^1/Z_2$ with four non-compact dimensions denoted by x^μ, $\mu = 0, \cdots, 3$. As we are interested in doubly warped model, the metric in this model can be chosen as

$$ds^2 = b^2(z)[a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2 \tag{1}$$

The angular coordinates y, z represent the extra spatial dimensions with moduli R_y and r_z respectively. The Minkowski metric in the usual 4-dimensions has the form $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. The functions $a(y), b(z)$ provide the warp factors in the y and z directions respectively. The total bulk-brane action of this model can be written as:

$$\begin{aligned}
S &= S_6 + S_5 + S_4 \\
S_6 &= \int d^4x dy dz \sqrt{-g_6}(R_6 - \Lambda), \\
S_5 &= \int d^4x dy dz \sqrt{-g_5}[V_1\delta(y) + V_2\delta(y-\pi)] \\
&\quad + \int d^4x dy dz \sqrt{-g_5}[V_3\delta(z) + V_4\delta(z-\pi)] \\
S_4 &= \int d^4x \sqrt{-g_{vis}}[\mathcal{L} - \hat{V}][\delta(y)\delta(z) + \delta(y)\delta(z-\pi) \\
&\quad + \delta(y-\pi)\delta(z) + \delta(y-\pi)\delta(z-\pi)]
\end{aligned} \tag{2}$$

Here, $V_{1,2}$ and $V_{3,4}$ are brane tensions of the branes located at $y = 0, \pi$ and $z = 0, \pi$, respectively. Λ is the cosmological constant in 6-dimensions. After solving Einstein's equations, the solutions to the warp factors of the metric as given in eq.(1) [2] are,

$$\begin{aligned}
a(y) &= \exp(-c|y|), \quad b(z) = \frac{\cosh(kz)}{\cosh(k\pi)} \\
\text{where } c &\equiv \frac{R_y k}{r_z \cosh(k\pi)}, \quad k \equiv r_z\sqrt{\frac{-\Lambda}{10M_P^4}}
\end{aligned} \tag{3}$$

Here, M_P is the 4-dimensional Planck scale. The 5-d RS model can be retrieved in the limit $r_z \to 0$. The warp factors $a(y)$ and $b(z)$ provide maximum suppression at $y = \pi$ and $z = 0$. For this reason we can interpret the 3-brane formed out of the intersection of 4-branes at $y = \pi$ and $z = 0$ as our standard model brane. The suppression on the standard model brane can be written as

$$f = \frac{\exp(-c\pi)}{\cosh(k\pi)} \tag{4}$$

The desired suppression of the order of 10^{-16} on the standard model brane can be obtained for different choices of the parameters c and k. However from the relation for c in eq. (3) it can be shown that if we want to avoid large hierarchy in the moduli R_y and r_z, the warping in one direction must be large while that in the other direction must be small. In our analysis we shall explore this feature for non-flat 3-branes over the entire parameter space of c, k and moduli ratio R_y/r_z for different values of brane cosmological constant.

II. Non-flat branes in multiply warped geometry model

Previously a generalization of the 5-dimensional RS model has been considered with the non flat 3-branes [6]. In this work we consider doubly compactified six dimensional space-time with Z_2 orbifolding along each of the compact direction. Thus the manifold under consideration is $[M^{(1,3)} \times S^1/Z_2] \times S^1/Z_2$ with four non-compact dimensions denoted by x^μ, $\mu = 0, \cdots, 3$. We choose a doubly warped general metric as:

$$ds^2 = b^2(z)[a^2(y) g_{\mu\nu} dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2 \tag{5}$$

Since orbifolding, in general, requires a localized concentration of energy we introduce 4-branes [(4+1)-dimensional space-time] at the orbifold fixed points namely at $y = 0, \pi$ and $z = 0, \pi$. The total bulk-brane action in this case is given by,

$$\begin{aligned} S &= S_6 + S_5 + S_4 \\ S_6 &= \int d^4x dy dz \sqrt{-g_6}(R_6 - \Lambda) \\ S_5 &= \int d^4x dy dz \sqrt{-g_5}[V_1 \delta(y) + V_2 \delta(y - \pi)] \\ &\quad + \int d^4x dy dz \sqrt{-g_5}[V_3 \delta(z) + V_4 \delta(z - \pi)] \\ S_4 &= \int d^4x \sqrt{-g_{vis}}[\mathcal{L} - \hat{V}][\delta(y)\delta(z) + \delta(y)\delta(z - \pi) \\ &\quad + \delta(y - \pi)\delta(z) + \delta(y - \pi)\delta(z - \pi)] \end{aligned} \tag{6}$$

The brane potential terms may be coordinate dependent as, $V_{1,2} = V_{1,2}(z)$ and $V_{3,4} = V_{3,4}(y)$. The term S_4 is the action for 3-branes located at $(y, z) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$.

The full 6-dimensional Einstein's equation can be written as,

$$\begin{aligned} -M^4 \sqrt{-g_6}(R_{MN} - \frac{R}{2} g_{MN}) &= \Lambda_6 \sqrt{-g_6} g_{MN} + \sqrt{-g_5} V_1(z) g_{\alpha\beta} \delta^\alpha_M \delta^\beta_N \delta(y) \\ &+ \sqrt{-g_5} V_2(z) g_{\alpha\beta} \delta^\alpha_M \delta^\beta_N \delta(y - \pi) \\ &+ \sqrt{-\tilde{g}_5} V_3(y) \tilde{g}_{\tilde{\alpha}\tilde{\beta}} \delta^{\tilde{\alpha}}_M \delta^{\tilde{\beta}}_N \delta(z) \\ &+ \sqrt{-\tilde{g}_5} V_4(y) \tilde{g}_{\tilde{\alpha}\tilde{\beta}} \delta^{\tilde{\alpha}}_M \delta^{\tilde{\beta}}_N \delta(z - \pi) \end{aligned} \tag{7}$$

Here M,N are bulk indices, α, β run over the usual four space-time coordinates (x^μ) and the compact coordinate z while $\tilde{\alpha}$, $\tilde{\beta}$ run over (x^μ) and the compact coordinate y. g, $\tilde{g}$ are the respective metrices in these (4+1)-dimensional spaces. For the metric (5), the different components of Einstein's equations reduce to a set of three independent equations,

$$\begin{aligned} {}^4G_{\mu\nu} + g_{\mu\nu}[\frac{3a'(y)^2}{R_y^2} + a(y)\frac{3a''(y)}{R_y^2} + \frac{2a(y)}{r_z^2}(3b'(z)^2 + 2b(z)b''(z))] \\ = -\frac{\Lambda_6}{M^4} a(y)^2 b(z)^2 g_{\mu\nu} \end{aligned} \tag{8}$$

$$6a'(y)^2 r_z^2 + a(y)^2[6R_y^2 b'(z)^2 + 4R_y^2 b(z)b''(z)] - (1/2)\ {}^4R\ b(z)^2 a(y)^2 R_y^2 r_z^2$$
$$= \frac{-\Lambda_6}{M^4} b(z)^2 a(y)^2 R_y^2 r_z^2 \quad (9)$$
$$6a'(y)^2 r_z^2 + 4r_z^2 a(y)a''(y) - (1/2)\ {}^4R\ r_z^2 R_y^2 a(y)^2 b(z)^2 10R_y^2 a(y)^2 b'(z)^2$$
$$= -\frac{\Lambda_6}{M^4} b(z)^2 a(y)^2 R_y^2 r_z^2 \quad (10)$$

${}^4G_{\mu\nu}$ and ${}^4R$ are the four dimensional Einstein tensor and Ricci scalar respectively, defined with respect to $g_{\mu\nu}$. Dividing both sides of the equation (8) by $g_{\mu\nu}$ for any μ, ν and rearranging terms it is seen that one side contains $a(y)$ and $b(z)$ and their derivatives, while the other side depends on the brane coordinates x^μ only. Thus we can equate each side to an arbitrary constant Ω such that,

$${}^4G_{\mu\nu} = -\Omega g_{\mu\nu} \quad (11)$$
$$a(y)^2[\frac{3}{R_y^2}\frac{a''(y)}{a(y)} + \frac{3}{R_y^2}\frac{a'(y)^2}{a(y)^2} + \frac{2}{r_z^2}$$
$$(3b'(z)^2 + 2b(z)b''(z)) + \frac{\Lambda_6}{M^4} b(z)^2] = \Omega \quad (12)$$

From equation(11) we identify Ω as the effective 4-D cosmological constant on the 3-branes.

III. De-Sitter brane ($\Omega > 0$)

We obtain the solutions for the warp factors $a(y)$ and $b(z)$ for de sitter brane from equation (12) as,

$$a(y) = \omega' \sinh\left[\ln\frac{c_2'}{\omega'} - cy\right], \quad b(z) = \frac{\cosh(kz)}{\cosh(k\pi)} \quad (13)$$

Where $c_2^{'}$ is an integration constant.
Here $\omega' = \omega\cosh(k\pi)$, with $\omega^2 = \frac{\Omega}{3k'^2}$, where $k^{'} = \sqrt{\frac{-\Lambda}{10M^4}}$, $k = k^{'} r_z$ and $c = \frac{R_y k}{r_z \cosh(k\pi)}$. Normalizing the warp factor to unity at the orbifold fixed point $y = 0$, we get $c_2' = \left[1 + (1+\omega'^2)^{\frac{1}{2}}\right]$. Note that the result for RS model generalized to six dimensions with flat branes is recovered in the limit $\omega \to 0$.
We now focus our attention to the boundary terms to determine the brane tensions. Using the explicit form of $a(y)$, $b(z)$ from equation(13) and implementing the boundary conditions across the two boundaries at $y = 0$, $y = \pi$ and $z = 0$, $z = \pi$ respectively, we obtain:

$$V_2(z) = 8M^2\sqrt{\frac{-\Lambda}{10}}\frac{(\frac{\omega'^2}{c_2'^2}e^{2c\pi} + 1)}{(\frac{\omega'^2}{c_2'^2}e^{2c\pi} - 1)}\text{sech}(kz) \quad (14)$$
$$V_1(z) = 8M^2\sqrt{\frac{-\Lambda}{10}}\text{sech}(kz)\frac{(1 + \omega'^2/c_2'^2)}{(1 - \omega'^2/c'^2)} \quad (15)$$

The above two equations imply that the two 4-branes sitting at $y = 0$ and $y = \pi$ have z dependent tensions. Similarly we find $V_3(y) = 0$ and $V_4(y) =$

$-\frac{8M^4k}{r_z}\tanh(k\pi)$. Thus we have determined the tensions for all the 4-branes in this model. The intersection of two 4-branes give rise to 3-brane. With this identification, the theory contains four 3-branes located at $(y,z) = (0,0), (0,\pi)$, $(\pi,0)$, (π,π). The metric on the 3-brane located at $(y=0, z=\pi)$ has no warping and can be identified with the Planck brane. Similarly we identify the standard model brane with the one at $y=\pi$, $z=0$ where the warping is maximum. Finally we obtain the expressions for 3-brane tensions in terms of ω, the induced brane cosmological constant as,

$$V_{vis} = 8M^2\left(\frac{\frac{\omega^2\cosh^2(k\pi)e^{2c\pi}}{(4+2\omega^2\cosh^2(k\pi))}+1}{\frac{\omega^2\cosh^2(k\pi)e^{2c\pi}}{(4+2\omega^2\cosh^2(k\pi))}-1}\right)\sqrt{-\frac{\Lambda}{10}} \tag{16}$$

$$V_{hid} = 8M^2\left[\left(\frac{1+\frac{\omega^2\cosh^2(k\pi)e^{2c\pi}}{(4+2\omega^2\cosh^2(k\pi))}}{1-\frac{\omega^2\cosh^2(k\pi)e^{2c\pi}}{(4+2\omega^2\cosh^2(k\pi))}}\right)\mathrm{sech}(k\pi)-\tanh(k\pi)\right]\sqrt{-\frac{\Lambda}{10}} \tag{17}$$

The two other 3-branes located at $(0,0)$ and (π,π) have tensions close to hidden brane and visible brane tensions respectively. Once again limit $\omega \to 0$ reproduces the expression for the 6-D flat 3-brane tensions[2] and with $r_z \to 0$ (i.e $k = k^{'} r_z \to 0$) we recover 5-D RS brane tensions[1]. Now to solve the gauge hierarchy problem we equate the warp factors at $y=\pi$, $z=0$ to the ratio of the mass scale in the two 3-branes given by 10^{-n} such that,

$$a(y)b(z)|_{y=\pi,z=0} = 10^{-n}$$
$$\omega'\sinh\left[\ln\frac{c_2'}{\omega'}-c\pi\right]\frac{1}{\cosh(k\pi)} = 10^{-n} \tag{18}$$

At this point, we keep n arbitrary but we will subsequently take it to be $\simeq 16$ to achieve a Planck to Tev scale warping.
Defining $c\pi = x$, the above equation has a positive root for e^{-x} as,

$$e^{-x} = \cosh(k\pi)\frac{10^{-n}}{c_2^{'}}\left[1+\left\{1+\omega^2 10^{2n}\right\}^{1/2}\right] \tag{19}$$

We re-parametrize the brane cosmological constant ω as: $\omega^2 \equiv 10^{-N}$.
Equation (18) now simplifies to:

$$-N = \frac{1}{\ln 10}\ln\left[\frac{4e^{-2x}-4\cosh(k\pi)10^{-n}e^{-x}}{\cosh^2(k\pi)[1+10^{-n}\cosh(k\pi)e^{-x}-2e^{-2x}]}\right] \tag{20}$$

This equation relates the three parameters present in this model, ω (which has been re-parametrized as 10^{-N}), k, and $x(=c\pi)$. The solution of x, derived from expression (19), is obtained as,

$$x = 16\ln 10 - \ln\cosh(k\pi) - \ln 2 + \ln[2+\frac{1}{2}10^{-N}\cosh^2(k\pi)] - \ln[1+\frac{1}{4}10^{-(N-2n)}] \tag{21}$$

Now from expression (21) we numerically calculate x i.e $c\pi$ for different values of $k\pi$ taking cosmological constant ω^2 as parameter for $n=16$. This value of n ensures the resolution of the gauge hierarchy problem for all the determined

values of the parameters in our subsequent analysis. Also we calculate the corresponding values of the ratio of two moduli R_y/r_z from the expression $c = \frac{R_y k}{r_z \cosh(k\pi)}$. The numerical values are given in the following table (1):

	$w^2 = 1$		$w^2 = 10^{-15}$		$w^2 = 10^{-40}$	
$k\pi$	$c\pi$	R_y/r_z	$c\pi$	R_y/r_z	$c\pi$	R_y/r_z
1.12	0.56	0.84	17.43	26.38	36.31	54.96
30.12	1.63×10^{-13}	0.032	5.24×10^{-6}	1.04×10^{6}	7.41	1.48×10^{12}

Table 1: Numerical values of $c\pi$ and R_y/r_z for every $k\pi$ and fixed cosmological constant ω(in Planck unit) in dS space-time

For given $k\pi$ the corresponding values of $c\pi$ and R_y/r_z saturate below $\omega^2 < 10^{-40}$. It may be seen from table(1), when the cosmological constant is very large (i.e approximately of the order of 1), for a small $k\pi \approx 1.12$, $c\pi$ is also small 0.56 and ratio of the two moduli is 0.84. Hence it is evident that for a very large cosmological constant we can have equal warping along both the extra dimensions and the two extra dimensional moduli are approximately close to l_{Planck} i.e. without any hierarchical values.
Now if we fix a large $k\pi = 30.12$, the value of $c\pi$ turns out to be very very small ($\approx 10^{-13}$) and also the ratio of the two moduli becomes hierarchical. From table (1), it may be noted that when we decrease the value of cosmological constant, for small $k\pi$, values of $c\pi$ become large and the two extra dimensional moduli are again reasonably close to each other with values close to l_{Planck}. But if we fix a large $k\pi$ then we can see that for decreasing ω^2, $c\pi$ values become small and the ratio of the two extra dimensional moduli become large.
Thus we conclude that a small but equal warping from both the extra dimensions and the minimum hierarchy between the two extra dimensional moduli can be achieved only when the induced cosmological constant on the brane is very large. But if we keep on decreasing the brane induced cosmological constant towards its present observed value which is estimated to be of the order of 10^{-120}, we cannot have equal warping along both the extra dimensions. Moreover if we demand that the two extra dimensional moduli are approximately of the same order, then in order to solve the gauge hierarchy problem the most favourable condition consistent with a small value of the brane cosmological constant is small $k\pi$ and large $c\pi$ i.e small warping along z direction and large warping along y direction. This resembles to 5-dimensional RS model perturbed slightly by the additional warping along z direction such that two 3-branes have scales close to Tev while the two other have scales close to Planck scale.

For Anti De-Sitter brane i.e $\Omega < 0$, we find that there is an upper bound on the brane induced cosmological constant similar to the one found in [6] which is 10^{-32} in Planck units. Repeating the entire analysis in the ADS sector for values of the cosmological constant lower than 10^{-32} we determine the correlations among the parameters illustrated in table (2) and (3).

$k\pi$	$w^2 = 10^{-40}$				$w^2 = 10^{-80}$			
	$c\pi$	R_y/r_z	$c\pi$	R_y/r_z	$c\pi$	R_y/r_z	$c\pi$	R_y/r_z
1.12	36.31	54.96	56.12	84.96	36.31	54.96	148.22	224.39
30.12	7.41	1.48×10^{12}	27.22	5.4×10^{12}	7.41	1.48×10^{12}	119.32	2.38×10^{13}

Table 2: Numerical values of $c\pi$ and R_y/r_z for different $k\pi$ with fixed cosmological constant ω in ADS space-time. The numerical values of ω are all in Planckian units

$k\pi$	$w^2 = 10^{-120}$			
	$c\pi$	R_y/r_z	$c\pi$	R_y/r_z
1.12	36.31	54.96	240.32	363.82
30.12	7.41	1.48×10^{12}	211.42	4.22×10^{13}

Table 3: Numerical values of $c\pi$ and R_y/r_z for different $k\pi$ with fixed cosmological constant ω in ADS space-time. The numerical values of ω are all in Planckian units

Here also we come to a similar conclusion that for small cosmological constant, in order to keep minimum hierarchy between the two extra dimensional moduli, the most favourable condition is small $k\pi \approx 1.12$ and large $c\pi \approx 36.31$.

IV. Conclusion

In this work the generalization of RS model has been done for a 6-dimensional ADS bulk with non-flat 3- branes. Requiring the warping from the hidden brane to visible brane $\sim 10^{-16}$ we find that for de-Sitter 3-brane, the warping along both the directions can be nearly equal with very small hierarchy between the two moduli only when the brane cosmological constant is very large compared to the present value. To achieve similar warping along both the directions for nearly vanishing cosmological constant we have to introduce large hierarchy between the moduli. On the contrary for small value of the brane cosmological constant with non-hierarchical small moduli $\sim l_{Planck}$, the warping along one direction (in our case along y) is very large while the other direction (i.e. z) is nearly flat. The corresponding values of the moduli c and k can be stabilized following the Goldberger-Wise stabilization mechanism [11] by introducing a 6-dimensional bulk scalar field. This leads to a scenario where the 3-branes can not have any intermediate scale and have energy scales either close to Tev or close to Planck scale. This remarkable correlations clearly point out that the most favoured condition for small cosmological constant, non-hierarchical moduli and the resolution of the gauge hierarchy problem correspond to a very large warping along y direction with very small warping along z-direction. This scenario is nothing but a weak perturbation of the original 5-dimensional RS model due to the presence of the sixth dimension which in turn leads to two 3-branes with energy scale close to Tev scale while two other 3-branes having energy scale close to Plank scale. Such a feature of brane clustering with closely-spaced energy

scales enhances when more and more extra warped dimensions are added leading to a stack of Tev scale 3-branes and a stack of Planck scale 3-branes. It has been shown in [3] that such scenario offers a possible geometric resolution of the fermion mass hierarchy problem among the standard model fermions. Our work thus explains that in a multiple warped geometry model the requirements of nearly flat 3-brane and non-hierarchical moduli lead naturally to a stack of closely clustered Tev 3-branes which in turn offers a geometric understanding of the fermion mass hierarchy as well as gauge hierarchy problem simultaneously.

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